THE JOURNAL OF
index investing
ETFs, ETPs & Indexing
SUMMER 2015 VOLUME 6 NUMBER 1 | www.IIJII.com

A Framework for Assessing Factors and Implementing Smart Beta Strategies

Jason Hsu, Vitali Kalesnik, and Vivek Viswanathan

A Framework for Assessing Factors and Implementing Smart Beta Strategies

Jason Hsu, Vitali Kalesnik, and Vivek Viswanathan

Jason Hsu
is the co-founder and vice chairman of Research Affiliates in Newport Beach, CA.
hsu@rallc.com

Vitali Kalesnik
is the head of equity research at Research Affiliates in Newport Beach, CA.
kalesnik@rallc.com

Vivek Viswanathan
is a senior researcher at Research Affiliates in Newport Beach, CA.
viswanathan@rallc.com

The assets invested in smart beta strategies have grown at a breathtaking pace,[1] as have the variety of smart beta products and the number of allegedly premium-bearing factors underlying smart beta indexes. Today, among the more reputable journals, one can find some 250 factors and, extrapolating from recent experience, one could expect that number to increase by 40 factors per year. In an earlier, simpler time—about 20 years ago—there were only five equity factors (the market, value, small-cap, momentum, and low beta factors). It is most unlikely that 250 factors are now driving equity returns. Indeed, given that some equity factors might have been behavioral in nature, while others are simply artifacts of historical data, one would actually expect the number of equity return factors to decline over time!

A number of finance researchers[2] argue that many of the recently discovered factors may be the results of data mining and thus unlikely to produce future excess returns. Indeed, with thousands of finance professors, doctoral candidates, and quantitative analysts running thousands of backtests and predictive regressions, the discovery of positive outliers is inevitable. Simply put, given the natural cross-sectional variance in returns, a portfolio strategy whose mean excess return is 0 with a tracking error of 4% has roughly a 5% chance of outperforming its benchmark by 1% in a 40-year backtest. Without careful robustness verifications, 1 in 20 portfolio simulations would accidentally look attractive.

A casual student of the empirical literature on factors might blithely mix a batch of factors to form a portfolio with multiple sources of excess return. Such a portfolio might well appear to have a Sharpe ratio greater than 2.0. Indeed, the more exotic and obscure a factor, the more valuable is its inclusion in the mix due to the low correlation in excess returns. However, if there is a meaningful probability that the factor is really just a data artifact, then including it is no different than adding casino bets to an investment portfolio. They, too, are uncorrelated with standard investment strategies; they, too, can have a run of positive outcomes that might fool less sophisticated individuals into believing that the odds are in the speculator's favor.

Thus, identifying an *actual* return factor in a zoo of factors, many of which are simply noise, is a critical step in selecting smart beta products that could deliver on the promise of long-term outperformance over traditional capitalization-weighted market beta.[3] In this article, we offer a practical framework to help investors separate the wheat from the chaff.

DETERMINING FACTOR ROBUSTNESS

Harvey et al. [2015] and Pukthuanthong and Roll [2014] offer stringent criteria for qualifying factors.[4] However, the average practitioner might find their statistical methods technically complex. We suggest a simple three-step heuristic for establishing the robustness of a factor premium. In our view, a robust factor is, first, one whose economic underpinnings and persistence have been debated and validated in numerous research papers published in top-tier journals. Second, the effect should persist across time periods and be statistically significant in most countries. Third, the effect should survive reasonable perturbations in the definition of the factor strategy. In the following subsections, we illustrate this validation framework by applying it to some of the more popular factors.

A Deep Literature Debating and Vetting the Factor

When a factor has been vigorously debated and vetted in the literature over a lengthy period, highly trained economists have thoroughly investigated the data and explored various economic rationales behind the existence and persistence of the factor premium. This process ensures that the effect is not a coding error and can be replicated by other researchers potentially using slightly different databases and construction methodologies. It is surprising how many published results cannot be replicated (Bailey et al. [2014 and 2015]).

While a lengthy literature surrounding a given factor does not necessarily guarantee a consensus on the origin or persistence of the premium, it does provide investors with a number of credible hypotheses to evaluate. Is the factor premium driven by risk or behavioral bias? If the latter, why might it persist? If there is no plausible explanation on the basis of risk or investor behavior, a dearth of follow-up literature will often reveal that a factor lacks a theoretical foundation. In this context, it is useful to understand how academic publishing works in general. Negative results are typically not published, even if they reject a previously reported factor—unless it is one of the classic factors such as smallcap.[5] Thus, the absence of vibrant follow-up research is a telltale sign that a purported factor has no real standing with financial researchers.

A scan of the existing literature finds many studies exploring the origin and application of factor strategies like value, momentum, low beta, and illiquidity; their existence does not appear to be in question. A search of the Social Science Research Network (SSRN) yields 2,306 hits for "value factor," 450 for "momentum factor," 260 collectively for "low-volatility factor" and "low beta factor," and 568 for "liquidity factor." These factors are debated and discussed to such an extent that we cannot attribute them to coding errors or one very particular definition of the factor.

Persistence Across Time and Geographies

Most published factors are "mined" from U.S. equity data, often with long-horizon datasets. It is worth examining whether the same factor premium can be observed in various sub-periods. For example, a substantial portion of the small-cap premium was concentrated in a few months in the 1930s; additionally, Horowitz et al. [2000] found that the small-cap premium has not delivered positive excess returns since the time of its discovery in the early 1980s. These sub-sample observations might meaningfully influence investors' beliefs regarding the magnitude and reliability of the small-cap premium.

Likewise, it is important to verify the existence of a factor in non-U.S. equity markets. If a factor is explained by risk and earns a risk premium in the U.S. dataset, one would expect to find that a similarly defined risk factor also commands a premium in other equity markets. If a factor is driven by persistent investor behavior in the United States, it would seem odd if non-U.S. investors were more rational and did not exhibit the same behavioral bias. When a factor that provides a positive premium in the United States does not earn a positive premium in other global markets, the factor is likely an artifact of the U.S. data rather than a reliable source of excess equity return. Thus one might treat the non-U.S. results as out-of-sample counter-evidence.

As illustrations, value and momentum appear to be robust in the non-U.S. dataset; indeed, they are stronger outside the United States (see Asness et al. [2013]). Similarly, Frazzini and Pedersen [2014] present evidence that the low beta anomaly is persistent across time, geographies, and asset classes, while Dimson and Hanke [2004] and Amihud et al. [2013] provide international evidence for the illiquidity premium. However, other popular factors such as smallcap and quality appear to struggle when tested more broadly in global equity datasets (see Hsu and Kalesnik [2014], and Beck and Kalesnik [2014]). Exhibit 1 replicates some of the abovementioned findings on global robustness for the popular factors.[6] For brevity and ease of exposition, we simply report the *p*-value from comparing the top 30% portfolio versus the bottom 30% portfolio corresponding to the specific factor definition. Reasonable variations to the portfolio cutoff do not produce meaningfully different conclusions. While the selection of countries and regions are by no mean comprehensive, they are illustrative of our framework for establishing out-of-sample robustness using non-U.S. data.

To define the value factor, we use the book-to-market ratio; to define the momentum factor, we use the total return in the period −2 to −12 months prior to portfolio formation; to define the low beta factor, following the methodology proposed by Frazzini and Pedersen [2014], we use the covariance of stock return and market return estimated over a five-year horizon, multiplied by the volatility over the past 12 months; to define the quality factor, we use ROE; to define the illiquidity factor, we use a ratio of adjusted daily volume and shares outstanding; to define the size factor, we use the company's recent market capitalization. To identify large and small companies for the size factor in the U.S., we use 50% of companies by count in the NYSE universe, and in markets outside of the U.S., we use 90% of companies by market capitalization. (The approach that we adopted for the U.S. is standard in the literature; the international sorting is similar to the U.S. methodology.) For all factors except size, we sort the universe of stocks into six portfolios: first by size into large and small, and then by the variable-defining factor into three portfolios comprising 30%, 40%, and 30%, respectively. Each of the portfolios is cap-weighted. For each factor, we average the extreme large and small portfolios and compare the Sharpe ratios of these long-only portfolios. Unless otherwise indicated in the table, the test is performed for the period 1967–2013 in the U.S. and 1987–2013 outside the U.S.

Perturbations in Definition

It should not surprise investors that researchers might consciously or inadvertently augment a factor construction until they achieve an optimal backtest—meaning one that shows the greatest adjusted alpha with the largest *t*-stat. This result is submitted to journals and published. Accordingly, it is a sensible practice to assume that most published results are inflated relative to the true factor premium. It is thus necessary to reverse this cherry-picking effect as much as possible.

One straightforward approach is to slightly perturb the definition of a factor. For example, the standard definition of the value characteristic is the book-to-market ratio (B/M). However, the trailing earnings-to-market (E/P) and dividend yield (D/P) ratios would be equally reasonable definitions. If factor portfolios

EXHIBIT 1
Factor Robustness Across Regions

Factor	Defintion	P-Value of Sharpe Ratio Difference (Significant if below 5%)	Significant (Yes/No)	Factor	Defintion	P-Value of Sharpe Ratio Difference (Significant if below 5%)	Significant (Yes/No)
Value	U.S.	0.10%	**Yes**	Quality	U.S.	6.17%	No
	U.K.	3.46%	**Yes**		U.K.	4.21%	**Yes**
	Europe ex U.K.	0.07%	**Yes**		Europe ex U.K.	1.95%	**Yes**
	Japan	0.00%	**Yes**		Japan	29.71%	No
Momentum	U.S.	0.00%	**Yes**	Illiquidity	U.S.	0.00%	**Yes**
	U.K.	0.00%	**Yes**		U.K. (1992–2013)	0.94%	**Yes**
	Europe ex U.K.	0.00%	**Yes**		Europe ex U.K.	4.35%	**Yes**
	Japan	28.86%	No		Japan (1992–2013)	11.83%	No
Low Beta	U.S.	0.00%	**Yes**	Size	U.S.	32.13%	No
	U.K.	0.11%	**Yes**		U.K.	58.19%	No
	Europe ex U.K.	0.02%	**Yes**		Europe ex U.K.	8.64%	No
	Japan	0.99%	**Yes**		Japan	5.52%	No

Source: Research Affiliates using CRSP/Compustat and Worldscope/Datastream data.

based on E/P and D/P produce zero or negative "value premiums" in the same dataset, this would cast serious doubt on the existence of the value anomaly and increase the suspicion that cherry picking drove the entire historical result. Indeed, we argue that the average across a meaningful set of perturbed definitions would be a better indication of what investors should expect to earn as the factor premium going forward.[7]

Among the popular factors, Fama and French [1992] found that measures such as earnings yield or dividend yield produce similar results as the book-to-market ratio for constructing the value factor. Jegadeesh and Titman [1993] showed that momentum is robust to various reasonable look-back formation and holding periods. Pastor and Stambaugh [2001] and Amihud [2002] show similar, positive illiquidity premiums for different measures of liquidity. Haugen and Heins [1975], Frazzini and Pedersen [2014], and Ang et al. [2006] have shown that the low-volatility factor survives definitional variations. On the other hand, Kalesnik and Kose [2014] found that quality shows very little robustness to perturbations in definitions, and Berk [1995] demonstrated that perturbations in the definition of size result in a loss of significance.

We illustrate the proposed perturbation exercise with some potential variations in factor definitions for the more popular factors in Exhibit 2. We encourage investors to attempt as many variations as necessary and reasonable to establish robustness as well as to estimate the true ex-ante magnitude of the factor premium.

The table indicates the variable that we used to define factors. For each factor (except size), we use 50% of companies by count in the NYSE universe to identify large and small companies. We sort the universe of stocks into six portfolios: first by size into large and small and then by the variable-defining factor into three portfolios representing 30%, 40%, and 30% respectively. Each of the portfolios is cap-weighted. Then we average the large and small extreme portfolios and compare the Sharpe ratios of these long-only portfolios. For size, we directly compare the Sharpe ratios of the cap-weighted portfolios consisting of large and small stocks identified using the variable indicated in the definition field of the table; for the Book and Asset variables we use NYSE 50% break points. The test is performed for the period of 1967–2013.

IMPLEMENTATION ISSUES

Active versus Passive Factor Implementation

Most studies report only the factor premiums measured from backtested paper portfolios, which ignore many details. The most important ones[8] are management or advisory fees and transaction costs. Both are direct and predictable components of returns. While

EXHIBIT 2

Factor Robustness to Perturbations in Definitions

Factor	Definition	P-Value of Sharpe Ratio Difference (Significant if below 5%)	Significant (Yes/No)	Factor	Definition	P-Value of Sharpe Ratio Difference (Significant if below 5%)	Significant (Yes/No)
Value	Book-to-Price	0.10%	**Yes**	Quality	Return on Equity	6.17%	No
	Earnings-to-Price	0.03%	**Yes**		Gross Profitability	17.89%	No
	Cashflow-to-Price	0.04%	**Yes**		Gross Margins	55.64%	No
	Dividends-to-Price	0.05%	**Yes**		Book Leverage	35.99%	No
Momentum	–2 to –12 Months	0.00%	**Yes**	Illiquidity	12 Month Turnover	0.00%	**Yes**
	–2 to –12 Months 3 mo. Hold	0.00%	**Yes**		6 Month Turnover	0.00%	**Yes**
	–2 to –6 Months	0.14%	**Yes**		2 Month Turnover	0.00%	**Yes**
	–1 to –12 Months	0.01%	**Yes**		ADV	0.00%	**Yes**
Low Beta	Low Beta	0.00%	**Yes**	Size	Breakpoint: NYSE 50th percentile	32.13%	No
	Low Volatility	0.00%	**Yes**		Breakpoint: NYSE 25th percentile	41.39%	No
	Low Beta 3 year	0.00%	**Yes**		Book	76.68%	No
	Low Volatility 3 year	0.00%	**Yes**		Assets	83.93%	No

Source: Research Affiliates using CRSP/Compustat and Worldscope/Datastream data.

these omissions are perfectly sensible for academic studies focused on understanding risk and behaviors in the asset pricing context, they could misrepresent the factor premium that investors could actually extract from a live portfolio. The recent rise in popularity for smart beta investing is in part related to the cost reduction provided by "indexation" of factor strategies. Smart beta strategies that are implemented in a passive, low-fee replication format can have a significant cost advantage over active management built on similar factor strategies, especially when the index vehicle is designed to curb turnover. Favorable management fees and low implied transaction costs are more likely to preserve premiums for the benefit of final investors instead of asset managers or brokers.

Asset management fees are readily available from asset managers, but transaction cost figures have been hard to come by. Novy-Marx and Velikov [2014] conducted what is probably the most comprehensive study of the impact of trading costs on factor portfolio returns. The study replicates a wide array of factors and computes transaction costs for these factors. Not surprisingly, Novy-Marx and Velikov find that factors with inherently low turnover—such as the market, low-volatility, and value factors—are not heavily affected by large transaction costs. These three factors can be comfortably implemented by means of the highly transparent index solutions typical of smart beta strategies.

For strategies associated with higher turnover, Novy-Marx and Velikov [2014] estimate transactions costs of 20 to 57 bps per month, enough to consume the whole factor premium. Among the factors we identified as robust, momentum and illiquidity are in the category where transaction costs are likely to be high. For these factors, a careful implementation, including measures to reduce transaction costs, is very important. In our view, for factor strategies that are high frequency and low capacity in nature, the transparent, formulaic approach that is the norm for passive implementation adds an additional burden. By disclosing their projected trades, passive momentum and illiquidity strategies essentially extend what amounts to a monthly invitation to front-runners. Certainly, exact replication of a formulaic momentum or illiquidity index, where liquidity management is more important given the nature of the strategy, would likely create unnecessary market price impact. Instead, active management provided by firms with superior trading skills (even market making skills) with moderate management fees and disciplined management of transactions costs may offer investors a better prospect to profit from the momentum and illiquidity factors. The unappealing alternative is to be the provider of premiums to diligent hedge fund managers looking to profit from arbitraging frequent or high-market-impact index rebalancing.

Smart Beta Index Construction

Choosing between active and passive investment management for a factor exposure is an important step in achieving efficient implementation. If a passive approach is deemed appropriate, then investors should next focus on studying in detail the implementation characteristics of the index of interest. For example, there are numerous ways to capture the value premium. Even a monkey randomly selecting stocks by throwing darts or a naïve equally weighted portfolio will snare the value premium no worse than many consciously value-oriented strategies.[9] However, the implementation details of various value strategies can be significantly different. Investors should look at the strategy's weighted average market capitalization for reassurance that a fund's capacity will not evaporate once it starts attracting assets. From the capacity perspective, the dart-throwing monkey, an equally weighted portfolio, or a book-to-price ratio-weighted index will have low investability.

Similarly, a strategy should have turnover that is just high enough to capture the premium, but no higher. For example, momentum requires monthly turnover; value calls for annual turnover. If a momentum strategy were only rebalanced once a year, it would leave most of the documented momentum premium on the table. If a value strategy were rebalanced more frequently than once a year, it would incur unnecessary trading costs but would not improve the premium capture. It is thus critical for investors to understand the appropriate rebalancing periodicity associated with a particular factor strategy when selecting smart beta indices for factor investing.

Both passive and active strategies can take measures to reduce transaction costs. In the passive domain, for example, Blitz et al. [2010] introduced the innovative concept of staggered rebalancing. This technique trades the portfolio in tranches, where each tranche adheres to the optimal rebalancing frequency for the target factor but trades on a different date. For example, a value portfolio can be broken into quarterly (or even monthly) tranches, each of which is rebalanced once a year, with the rebalances occurring at three-month (or one-month) intervals. Staggered rebalancing ensures that, at each rebalance, the dollar volume traded is lower, and therefore the market impact is lower.

There are other techniques to lower turnover. For example, passive managers can use banding to reduce the unproductive turnover that arises from trading stocks on the margins of their selection criteria. This will meaningfully reduce index reconstitution turnover on each rebalancing date. Of course, skilled active and passive managers have a whole repertoire of measures where, by mindful execution, they might not only minimize the market impact but also bring positive value to the final investors by providing liquidity and capturing a portion of the bid-ask spread.

FACTOR ALLOCATION IN A PORTFOLIO

Having identified a set of robust factors and products that implement them efficiently, investors still need to determine how they can best combine the chosen factors in the portfolio. Here, consultants and registered investment advisors (RIAs) experienced in the logic and methods of asset allocation can leverage the same experience to help investors construct portfolios in view of the investors' financial objectives. Unsurprisingly, it appears that the appropriate factor allocation will be highly dependent on the investor's definition of risk, her risk tolerance, her ability to implement tactical/dynamic allocation, and the governance structure and politics at her organization.

When considering factor allocations, investors should decide what type of risk they are most sensitive to: absolute risk (volatility) or risk relative to the benchmark (tracking error). An example will illustrate why this distinction is important. At first blush, the low-volatility factor sounds very attractive. After all, who wouldn't want a similar or even higher return at a lower total risk? But the higher return from low-volatility investing comes with higher tracking error. If in the next five years the general market were to go up by 20% per year, low-volatility strategies will predictably underperform on average by 6% per annum and could easily underperform by 10% per annum—they have market beta significantly below unity (roughly 0.7 on average) and annualized tracking error on the order of 10%. With this magnitude of potential underperformance, does low-volatility investing still sound like a good idea? For pension funds with a risk budget measured in tracking error, this risk profile might be completely unacceptable. This hypothetical example brings out the fact that low-volatility strategies are more likely to benefit investors if the governance structure allows them to ignore benchmark performance with impunity. A low-volatility smart beta strategy may be a great choice for investors who can disregard tracking error risk because they care only about absolute risk. If tracking error is a concern, the investor will have to look to other strategies. Only after investors have determined the type and amount of risk they are willing to take does the question of how to best combine factors make sense.

As is the case with allocating to asset classes, correlations, volatilities, and expected returns play critical roles in helping investors formulate the appropriate allocation. Historical average returns, correlations, and volatilities are easy to compute. This information, however, may be insufficient to make an efficient allocation decision, and, when used carelessly or ineptly, can be misleading and counterproductive to the exercise. DeMiguel et al. [2006] show that mean-variance optimized portfolios based on historical sample averages and correlations underperform naïve equal weighting as an allocation strategy; that is, the investor might actually be better off ignoring historical data in formulating asset allocation strategies.

The challenge with applying the traditional optimization approach in modern portfolio allocation is that expected returns and correlations can be time-varying and are often even mean-reverting. Asness et al. [2000] and Cohen et al. [2003] have shown that the premium associated with the value factor is prone to vary over time and can be predicted by the spread in valuation levels between the value and growth portfolios. Thus, premium and correlation estimates that are heavily influenced by recent data might actually be extremely misleading for asset allocation decisions.

As finance researchers continue to discover new facts, the prospect for achieving meaningfully better results than naïve equal allocation has also improved. We are constantly learning about new conditioning variables, which can help us estimate forward return and risk parameters for factors. For example, Barroso and Santa-Clara [2014] and Daniel and Moskowitz [2013] have shown that the probability for a momentum factor crash increases substantially when market volatility spikes. This finding tells us that (downside) volatility for the momentum factor can be predicted better by looking at market volatility (as inferred from the CBOE Volatility Index, known as VIX) than by looking at recent momentum strategy volatility.

Invariably, the capacity and access cost of different factor strategies will also play an important role in setting the appropriate allocation policy for an investor. Low-capacity strategies like illiquidity or potentially momentum are unlikely to be dominant portions of the portfolio core regardless of how uncorrelated they might be with the other factor strategies or how high their paper-portfolio Sharpe ratio might appear. In the current environment with increasing fee sensitivity, smart beta/factor products with high expense ratios are also unlikely to garner large allocations for optics and other reasons. Investors and their consultants would certainly be far more able to assess the impact of these two considerations on their allocation given their portfolio size and governance structure than any theoretical prescription coming out of academic models.

IN CLOSING

Factor investing as a theoretical concept sounds simple, but the literature on risk factors is littered with more than 250 supposed factors, and more are reported every year. Add to these the combinations of factors advertised by various providers, and the choice becomes nigh impossible. The truth is the vast majority of these factors will not produce a reliable positive premium in the future. They are likely data-mined artifacts from historical equity data.

We propose a framework for identifying robust and investable factors that can be incorporated into smart beta strategies. For a factor to be considered robust, it must be based on a meaningful economic intuition, be supported by deep empirical literature, be robust across timespans and geographies, and deliver excess returns despite minor changes in definition. For a factor to be considered passively implementable, it must deliver excess returns in liquid names, require only infrequent trading and low turnover, and have the capacity to accommodate very large in- and outflows. Otherwise, highly skilled (and thus more costly) active trading would be required for effectively capturing the premium.

We also suggest that factor allocation shares many of the same challenges as traditional asset allocation. The time-varying nature of expected returns, volatilities, and correlations often makes historical sample estimates misleading and even counterproductive in the portfolio optimization exercise. Furthermore, investor risk preference and definition are sufficiently idiosyncratic as to make one-size-fits-all advice less appropriate.

ENDNOTES

[1]Hortense Bioy [2014] writes, "Global assets in strategic beta ETFs have increased by 87% in two years and now account for $380 billion, according to Morningstar data."

[2]See Harvey et al. [2015], Harvey and Liu [2014], Pukthuanthong and Roll [2014], McLean and Pontiff [2015], and Bailey et al. [2014 and 2015].

[3]We borrow the "zoo of factors" from John Cochrane's 2011 AFA presidential address.

[4]Harvey, Liu, and Zhu adjust the threshold of statistical significance for the fact that many researchers are constantly seeking to identify *new* factors. Pukthuanthong and Roll offer an interesting protocol for identifying a set of factors by connecting cross-sectional factor returns to the covariation of returns; their approach is supposed to capture variations in the real economy. The issues tackled in both these article are extremely interesting from an academic perspective. Nonetheless, in this article we address a different problem: Which of the reported factors are likely to benefit long-only passive investors?

[5]Shumway and Warther [1999] find that the small-cap premium most likely originated with mistakes in the treatment of delisting returns for small-cap stocks.

[6]For our replication, we use the most common factor definition found in the literature. We will further examine other equivalent definitions in later tables to examine if results change meaningfully.

[7]This is the very same intuition behind using shrinkage estimates in statistics.

[8]Another significant detail may be the accuracy of delisting returns. Most databases have delisting returns that are unrealistically high relative to what an investor would actually be able to obtain on the over-the-counter market once a stock is dropped from a major exchange. As noted, Shumway and Warther (1999) demonstrated that this bias is probably the essential reason why we observe a size factor in paper portfolios simulated using U.S. data.

[9]Arnott et al. [2013] studied a number of strategies that are not price-weighted, and discovered that a simulated dart-throwing monkey, equally weighted portfolios, and a large variety of random portfolios perform no worse than many

popular smart beta strategies. The "monkey" strategy, as well as the smart beta strategies studied in the paper, outperformed cap-weighted benchmarks due to their exposure to the value factor. This empirical observation is consistent with Berk's [1995] theoretical implication that the value effect should be present in any non-price-weighted strategy.

REFERENCES

Amihud, Y. "Illiquidity and Stock Returns: Cross-Section and Time-Series Effects." *Journal of Financial Markets*, 5 (2002), pp. 31-56.

Amihud, Y., A. Hameed, W. Kang, and H. Zhang. "The Illiquidity Premium: International Evidence." Working Paper. Available at SSRN: http://ssrn.com/abstract=2191949 (July 16, 2013).

Ang, A., R.J. Hodrick, Y. Xing, and X. Zhang. "The Cross-Section of Volatility and Expected Returns." *Journal of Finance*, 61 (2006), pp. 259-299.

Arnott, R.D., J. Hsu, V. Kalesnik, and P. Tindall. "The Surprising Alpha from Malkiel's Monkey and Upside-Down Strategies." *The Journal of Portfolio Management*, 39 (2013), pp. 91-105.

Asness, C.S., J.A. Friedman, R.J. Krail, and J.M. Liew. "Style Timing: Value versus Growth." *The Journal of Portfolio Management*, 26 (2000), pp. 50-60.

Asness, C.S., T.J. Moskowitz, and L.H. Pedersen. "Value and Momentum Everywhere." *Journal of Finance*, 68 (2013), pp. 929-985.

Bailey, D.H., J.M. Borwein, M. Lopez de Prado, and Q.J. Zhu. "Pseudo-Mathematics and Financial Charlatanism: The Effects of Backtest Overfitting on Out-of-Sample Performance." Notices of the American Mathematical Society, 61 (2014), pp. 458-471. Available at SSRN: http://ssrn.com/abstract=2308659.

——. "The Probability of Backtest Overfitting." Available at SSRN: http://ssrn.com/abstract=2326253 (February 23, 2015).

Barroso, P., and P. Santa-Clara. "Momentum Has Its Moments." *Journal of Financial Economics*, forthcoming. Available at SSRN: http://ssrn.com/abstract=2041429 (December 10, 2014).

Beck, N., and V. Kalesnik. "Busting the Myth of Size." Research Affiliates (2014).

Berk, J.B. "A Critique of Size-Related Anomalies." *Review of Financial Studies*, 8 (1995), pp. 275-286.

Bioy, H. "Are Strategic Beta ETFs a Threat to Active Fund Managers?" Morningstar ETF Research & Insights (November 18, 2014).

Blitz, D., B. Van der Grient, and P. Van Vliet. "Fundamental Indexation: Rebalancing Assumptions and Performance." *The Journal of Index Investing*, 1 (2010), pp. 82-88.

Cochrane, J.H. "Presidential Address: Discount Rates." *Journal of Finance*, 66 (2011), pp. 1047-1108.

Cohen, R.B., C. Polk, and T. Vuolteenaho. "The Value Spread." *Journal of Finance*, 58 (2003), pp. 609-642.

Daniel, K.D., and T.J. Moskowitz. "Momentum Crashes." Swiss Finance Institute Research Paper No. 13-61; Columbia Business School Research Paper No. 14-6; Fama–Miller Working Paper. Available at SSRN: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2371227 (December 24, 2013).

DeMiguel, V., L. Garlappi, and R. Uppal. "1/N." EFA 2006 Zurich Meetings. Available at SSRN: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=911512 (June 22, 2006).

Dimson, E., and B. Hanke. "The Expected Illiquidity Premium: Evidence from Equity Index-Linked Bonds." *Review of Finance*, 8 (2004), pp. 19-47.

Fama, E.F., and K.R. French. "The Cross-Section of Expected Stock Returns." *Journal of Finance*, 47 (1992), pp. 427-465.

Frazzini, A., and L.H. Pedersen. "Betting against Beta." *Journal of Financial Economics*, 111 (2014), pp. 1-25.

Harvey, C.R., and Y. Liu. "Backtesting." Available at SSRN: http://ssrn.com/abstract=2345489 (October 7, 2014).

Harvey, C.R., Y. Liu, and H. Zhu. "…And the Cross-Section of Expected Returns." Available at SSRN: http://ssrn.com/abstract=2249314 (February 3, 2015).

Haugen, R.A., and A.J. Heins. "Risk and the Rate of Return on Financial Assets: Some Old Wine in New Bottles." *Journal of Financial and Quantitative Analysis*, 10 (1975), pp. 775-784.

Horowitz, J.L., T. Loughran, and N.E. Savin. "Three Analyses of the Firm Size Premium." *Journal of Empirical Finance*, 7 (2000), pp. 143-153.

Hsu, J., and V. Kalesnik. "Finding Smart Beta in the Factor Zoo." Research Affiliates (2014).

Jegadeesh, N., and S. Titman. "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency." *Journal of Finance*, 48 (1993), pp. 65-91.

Kalesnik, V., and E. Kose. "The Moneyball of Quality Investing." Research Affiliates (2014).

McLean, R.D., and J. Pontiff. "Does Academic Research Destroy Stock Return Predictability?". AFFI/EUROFIDAI, Paris December 2012 Finance Meetings Paper. Available at SSRN: http://ssrn.com/abstract=2156623 (February 17, 2015).

Novy-Marx, R., and M. Velikov. "A Taxonomy of Anomalies and Their Trading Costs." Working Paper No. w20721, National Bureau of Economic Research (2014).

Pastor, L., and R.F. Stambaugh. "Liquidity Risk and Expected Stock Returns." Working Paper No. w8462, National Bureau of Economic Research (2001).

Pukthuanthong, K., and R. Roll. "A Protocol for Factor Identification." Available at SSRN: http://ssrn.com/abstract=2342624 (January 27, 2014).

Shumway, T., and V.A. Warther. "The Delisting Bias in CRSP's Nasdaq Data and Its Implications for the Size Effect." *Journal of Finance*, 54 (1999), pp. 2361-2379.