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Under neoclassical finance theory, a trade can change the price of a security only if it contains information (e.g., if it is initiated by an investor attempting to exploit some private knowledge) or if it contains no information but cannot be distinguished by market participants from information-carrying trades (Kyle [1985]). Although this approach provides interesting insights, other theoretical and empirical works suggest that even trades that are known to be uninformed can have clearly observable, economically significant market impact (Shleifer [1986]; Schultz [2008]; Madhavan [2000, 2003]).

In this article, we distinguish between explicit and implicit costs, give an overview of a proposed linear model, and explain the factors to which the model attributes implicit costs. We then set out the essential mathematics of the model before proceeding to take net investment flows and the frequency of index rebalancing into account. We also demonstrate the logical relationship between the market impact cost and the conventional indicators of liquidity, market capitalization and turnover. Finally, we use the model to compare the market impact costs of rebalancing nine portfolios (a market-capitalization-weighted index, an equal-weighted index and a fundamentally weighted index in each of three geographical regions). We find that, relative to a broad cap-weighted U.S. index, the market impact costs are progressively higher for fundamentals-weighted and equal-weighted indexes in the United States, developed markets excluding the United States and emerging markets.

EXPLICIT AND IMPLICIT TRADE COSTS

In its most flexible form, an investment strategy can make trading decisions continuously, based on the information available at each instant of time. However, the associated complexity, operation and monitoring costs, and risks are high. Partially as a result of these considerations, less flexible but more transparent and lower-cost index-based strategies have grown in popularity among individual as well as institutional investors.

There are three main reasons for an index-tracking portfolio manager to trade: 1) to reflect index reconstitution or rebalancing,[1] 2) net investment flows and 3) corporate actions. In our analysis, we ignore corporate action trading because it is a relatively small part of the overall trading and because the pattern of such trades is highly variable.

The implementation cost of an index-based strategy can be decomposed into two components: explicit and implicit. The explicit component, often referred to as the "implementation tracking error," is the observed difference between the performance of the fund and that of the index.[2] The implicit component is the unobserved reduction in the performance of the underlying index due to trading activity.

The relative contributions of the two components will depend on the implementation strategy the tracking fund adopts. For example, consider a fund that replicates the index by trading market-on-close. Assuming the index is calculated using the same closing prices, there is no performance gap between the index and this particular fund, apart from fees (brokerage commissions, transaction-related custodial charges and so on) incurred by or allocated to the fund. Most of the implementation cost is, therefore, contained in the implicit component.

Alternatively, consider a fund that spreads all its trades over a long time period to minimize the price impact. In this case, the implicit component may be very small. However, the fund will not trade at the same time (and, therefore, at the same prices) as the index and thus will have a tracking error against the index. Here, most of the implementation cost is represented by the explicit component.

This article focuses on the implicit component. We assume a simplified theoretical model of price impact and use it to discern unobvious and nontrivial characteristics of implicit implementation costs.

Further complexity arises when an index strategy is followed by multiple managers. In this case, the same securities are traded by several market participants, often using different implementation tactics. We abstract from this complexity by modeling the impact of the aggregate trading of all managers tracking a given index rather than the impact of each individual manager.

INTRODUCING A LINEAR MARKET IMPACT MODEL

Although our framework can be modified to accommodate a different functional form, we chose a linear market impact model:

$$\frac{\Delta p}{p} = k\frac{X}{V} \tag{1}$$

Here, p is the pre-trade price of a stock, k is a constant that may depend on the individual market, X is the size of the trade in dollars (greater than zero for a buy and less than zero for a sell), V is the aggregate volume across all the company's share classes in dollars and Δp is the price change, that is, the arithmetic difference between the stock's post-trade and pre-trade prices. We are aware that the industry often uses models where the price impact is proportional to the square root of the normalized trade size; however, our internal empirical research indicates that a linear model provides a good fit for index rebalance trading when the trade sizes are less than the average daily volume (ADV). It is possible that the index trading is more expensive than trades made by a single investor due to the lack of coordination between traders. For instance, if a significant fraction of the ADV is traded by a single investor, the trade might be spread carefully over a period of time. If the same occurs in the index rebalance context, each individual implementer may not see the trade size as being unusually large and hence cluster the trades around the market close.

A variety of market impact models have been used in the literature. A clear distinction is made between permanent and temporary market impact costs, with the latter typically assumed to decay over a period less than an hour. Almgren [2005] and Huberman and Stanzl [2004] strongly argued in favor of a linear market impact for permanent costs. Gatheral [2008] also supported a linear model for the temporary impact. However, Huberman and Stanzl [2004] held that temporary price impacts can take a more general form, and Almgren [2005] advocated a concave function for temporary market impacts.

The distinction between permanent and temporary market impact is less clear when it comes to the large-scale trading with which we are concerned in this article. The market impact from an index rebalance takes at least several days to decay, and, due to noise, event studies are limited in their ability to separate permanent from temporary impact. In addition, over a longer term, even "permanent" market impact may decay. Our research is in line with the approach adopted by Johansson and Pekkala [2013], who developed a measure of relative portfolio capacity.

DECOMPOSING THE MARKET IMPACT

Note that if the price impact is linear in the size of the trade, then the monetary cost is quadratic in the size of the trade. Suppose a security trades at price p. The trader buys X dollars of shares, causing the price to go up by $\Delta p = p\frac{kX}{V}$. Let us assume that the average execution price is increased by the same amount.[3] This increase in price translates into the increased amount paid for the purchase equal to $X\frac{\Delta p}{p} = \frac{kX^2}{V}$. We treat this amount as an implementation cost because we assume that over the investor's holding horizon it will disappear.

In order to calculate the total implicit implementation cost from a rebalance, we need to sum up these costs across all the stocks traded at rebalance. Under some simplifying assumptions, the implicit cost of rebalancing is a function of five factors: base impact, rebalance frequency, coverage, effective turnover and tilt:

$$\text{Cost} = k\frac{\text{Base impact} \times \text{Effective turnover} \times \text{Tilt}}{\text{Coverage} \times \text{Rebalance frequency}} \qquad (2)$$

The *base impact* factor is the ratio of the assets under management in a given strategy to the dollar value of shares traded daily across all the stocks in the universe of interest, scaled by a constant factor. The key observation is that, under our assumptions, the cost is linear in strategy size.

The *effective turnover* factor reflects the fact that if there were no trading, there would be no cost. An important observation is that trading additions and deletions to the index are, on average, much more expensive than trading stocks that remain in the index.

$$\text{Effective turnover} = \text{Replacement turnover} + \text{Adjusted reweighting turnover}^2$$

As a result, the effective turnover is a sum of two terms. The first term represents the turnover generated by additions and deletions, and it enters the expression directly.[4] The second term represents the turnover generated by reweighting existing constituents at rebalance and by trading intraperiod net investment flows; it enters the expression squared. (The second term is adjusted by a factor that depends on how evenly the turnover is distributed across the liquidity available in the portfolio).

Tilt indicates how far the portfolio departs from the volume-weighted index. A volume-weighted index has the lowest implementation cost because it essentially utilizes all the available volume to the greatest degree. Index tilt is defined as the weighted-average ratio of the actual weight to the volume weight across all securities.

The smallest possible value of tilt is 1, and this level is achieved only by a volume-weighted portfolio. Logically, because the tilt equals 1 plus the dispersion of the portfolio to volume-weight ratios, it reaches its minimum value only when there is no dispersion in ratios, and if all ratios are the same, then the portfolio weights are proportional to the volume weights. Given that weights, by definition, sum to 1, it follows—in this instructive but unlikely case—that the portfolio and volume weights would be identical. In more realistic cases, if each security is randomly overweighted or underweighted relative to the volume weight by z% (i.e., its weight is set to either $v_i[1 + z]$ or $v_i[1 - z]$), the resulting portfolio will have tilt of about $1 + z^2$. Tilt is especially sensitive to the portfolio weights of securities with very small relative volume.

Coverage is the ratio of the total trading volume of the index constituents to the total trading volume of the entire universe. The coverage is 1 for a portfolio that contains every stock in the universe; it can fall by an order of magnitude or more for an index that consists mostly of smaller stocks or one that contains very few stocks.

Finally, *rebalance frequency* can significantly affect the implicit cost. For example, an index with a quarterly rebalance will experience implicit intraperiod costs four times lower than an annually rebalanced index if the turnover and other characteristics are the same between the two indexes.

The rebalance frequency relevant in this calculation is that at which each stock, rather than the entire index, is traded over the course of the year. This distinction is critical. Consider a strategy that covers Asia, Europe, America and Africa, and that trades Asia on June 30, Europe on September 30, America on December 30 and Africa on March 30. Although this strategy executes trades four times a year, each individual stock is traded only once a year (ignoring the small trade to bring the regions themselves back to the desired weights). For this strategy, the implementation cost is the same as for the strategy traded once a year.

This analysis assumes that trades are sufficiently separated in time so that the price impact from the previous rebalance does not affect the current rebalance trade. It is a reasonable assumption for index-based portfolios, because they rarely rebalance more frequently than once a month.

With this overview in mind, let us turn to the mathematics of the market impact model.

CALCULATING THE MARKET IMPACT OF REBALANCE TRADING

Consider an annually rebalanced investment strategy implemented by trading to the precise target weights on the day of the rebalance. The market impact cost incurred from trading a single security s equals $k\frac{(A\Delta w_S)^2}{V_S}$, where k is the coefficient in the linear price impact relationship; Δw_S is the change in the weight of the stock at rebalance; V_S is its average daily trading volume (ADV); and A is the amount of assets invested in the strategy. The impact on the index, measured as a percent reduction in index returns, is then

$$C = \frac{1}{A}\sum_{S\in P\cup D} k\frac{(A\Delta w_S)^2}{V_S} \tag{3}$$

Here P is the portfolio as it stands after the rebalance; D is the set of stocks deleted at rebalance.

For ease of exposition, we will initially focus on the stocks that will remain in the portfolio after the rebalance. Later, we will account for the impact of the deleted stocks. We can rewrite the cost as follows:

$$C_P = k\frac{A}{V_U}\frac{V_U}{V_P}\sum_{S\in P}\frac{\Delta w_S^2}{v_S} \tag{4}$$

V_U is the sum of ADV of all the securities in an arbitrary universe[5] U; V_P is the like sum for the securities in portfolio P; and $v_S = \frac{V_S}{V_P}$ is the volume weight of stock s in portfolio P.

Let us denote δ_S the fraction of the security's position traded: $\delta_S = \frac{|\Delta w_S|}{w_S}$, where w_S is the stock's weight in P. Note that δ_S can exceed 1.

We can then write

$$C_P = k\frac{A}{V_u}\frac{V_u}{V_p}\sum_{S\in P} w_S\delta_S^2\frac{w_S}{v_S} \tag{5}$$

We assume for simplicity that δ_S^2 is uncorrelated with $\frac{w_S}{v_S}$, and write

$$C_P = k\frac{A}{V_u}\frac{V_u}{V_p}\sum_{S\in P} w_S\delta_S^2\sum_{S\in P} w_S\frac{w_S}{v_S} \tag{6}$$

This assumption is reasonable when the turnover is not very high. On the flip side, when the turnover is very high, the asymmetry between buys and sells weakens this approximation. For example, if a purchase moved a security's weight from 1% to 5%, then the fraction traded is 4; if the security is sold and its weight moves from 5% to 1%, the fraction traded is 0.8. Thus, we can expect that buys have higher average δ_S than sells do.

Let us connect the cost to the turnover at rebalance. The total two-way turnover can be decomposed into parts due to the additions, deletions and reweighting: $T = T_a + T_d + T_r = T_a + T_d + \sum_{S\in R} w_S|\delta_S|$, where r is the set of stocks retained at rebalance.

We can decompose

$$\sum_{S\in P} w_S\delta_S^2 = T_a + \sum_{S\in R} w_S\delta_S^2 \tag{7}$$

Furthermore, let us denote the ratio of the average squared δ_S to the square of the average absolute value of δ_S at rebalance as $\psi_r = \frac{E^w[\delta_S^2]}{(E^w[|\delta_S|])^2}$. The value ψ_r depends on the distribution of rebalance trades. For example, if all trades were the same fraction of the security's weight, then $\psi_r = 1$; if the target weights were the same year to year and the trades were driven exclusively by price drift,[6] then $\psi_r \approx \frac{\pi}{2}$.

Note that

$$\psi_r = \frac{E^w\left[\delta_S^2\right]}{\left(E^w\left[|\delta_S|\right]\right)^2} = \frac{\sum_{S\in R} w_S\delta_S^2 \Big/ \sum_{S\in R} w_S}{\left(\sum_{S\in R} w_S|\delta_S| \Big/ \sum_{S\in R} w_S\right)^2}$$

$$= \frac{\sum_{S\in R} w_S\delta_S^2}{\left(\sum_{S\in R} w_S|\delta_S|\right)^2}\sum_{S\in R} w_S = \frac{\sum_{S\in R} w_S\delta_S^2}{\left(\sum_{S\in R} w_S|\delta_S|\right)^2}(1-T_a)$$

and, therefore,

$$\sum_{S\in P} w_S\delta_S^2 = T_a + \sum_{S\in P} w_S\delta_S^2 = T_a + \frac{1}{1-T_a}\psi_r T_r^2$$

This allows us to rewrite the cost expression as

$$C_P = k\frac{A}{V_u}\frac{V_u}{V_p}\left(T_a + \frac{1}{1-T_a}\psi_r T_r^2\right)\sum_{S\in P} w_S\frac{w_S}{v_S} \tag{8}$$

Thus far, we have ignored deletions. As a modest simplification, we assume that deletions contribute to the total performance impact in the same proportion as additions and that the weighted average of $\frac{w_S}{v_S}$ is the same for the deleted stocks as for the stocks in the portfolio. Therefore, we write the total performance cost as

$$C = k\frac{A}{V_u}\frac{V_u}{V_p}\left(T_a + T_d + \frac{1}{1-T_a}\psi_r T_r^2\right)\sum_{S\in P} w_S \frac{w_S}{v_S} \quad (9)$$

Note that the last term on the right-hand side can be rewritten as follows:

$$\sum_{S\in P} w_S \frac{w_S}{v_S} = 1 + \mathrm{var}^v[\frac{w_S}{v_S}] \quad (10)$$

Here $var^v[x] \equiv \sum_S v_S(x_S - \sum_S v_S x_S)^2$ is a volume-weighted variance. This can be interpreted as the variance of a random variable that takes values from the set $\{x_S\}$ with probability of choosing each of those values equal to v_S.

CAPTURING THE EFFECT OF REBALANCING FREQUENCY

We will now extend this expression to accommodate multiple rebalance dates per year. If a trade in a single stock is spread equally over N periods, the linear price impact model suggests that the total performance cost in a given period will drop by N times. More generally, if the stock is traded in monetary amounts $q_1, q_2, \ldots, q_N$, the total cost due to the market impact will equal $\frac{k}{v}\Sigma_i q_i^2$. Compared with the cost of trading the entire amount $Q = \sum_i q_i$ on a single day, the trading cost changes by the factor $\phi = \frac{\Sigma_i q_i^2}{(\Sigma_i q_i)^2}$; this factor represents how well the trades are spread over time. For trades concentrated on a single day (e.g., $q_1 = Q$, $q_2 = 0$, $q_3 = 0, \ldots,$), $\phi = 1$. If all the trades are equal in size, $\phi = \frac{1}{N}$, where N is the number of trades.

Denoting the adjustment for the frequency of trading ϕ_r, the expression for the total performance impact will change to

$$C = k\phi_r\frac{A}{V_u}\frac{V_u}{V_p}\left(T_a + T_d + \frac{1}{1-T_a}\psi_r T_r^2\right)\sum_{S\in P} w_S \frac{w_S}{v_S} \quad (11)$$

INCORPORATING INVESTMENT FLOWS

To facilitate investigating the impact of net investment flows on performance, we will make a strong assumption. Normally, an index may be implemented by numerous funds, at least some of which will have daily flows. Because our price impact model is intended to work only when trades are separated in time by at least a month, we will assume that all implementations of the strategy allow investments only at month- or quarter-end. Denote T_f the aggregate amount of strategy-wide trading (across all the index-tracking funds) due to net investment flows. Investment flows result in proportional trades for each stock; however, the absolute amounts of flows are not constant throughout the year. The contribution of investment flows to the market impact of the index is straightforwardly represented by adding one more term to the turnover component:

$$C = k\frac{A}{V_u}\frac{V_u}{V_p}\left(\phi_r[T_a + T_d + \frac{1}{1-T_a}\psi_r T_r^2] + \phi_f T_f^2\right)\sum_{S\in P} w_S \frac{w_S}{v_S} \quad (12)$$

Here, $\phi_f = \frac{\Sigma_i f_i^2}{(\Sigma_i |f_i|)^2}$ (where f_i is the net investment flow at time i) is the measure of the dispersion of the investment flows.

INCORPORATING WAMC

Traditionally, investability, as indicated by the implicit trading costs of a strategy, is evaluated using turnover and weighted-average market capitalization (WAMC).[7] These measures are often compared across different strategies, and the strategy with the lower turnover and higher WAMC is assumed to have lower trading costs. However, this common-sense approach will not suffice when investment approaches or universes differ meaningfully. Our framework allows such comparisons; in addition, it demonstrates that there are several other portfolio characteristics besides WAMC and turnover that affect investability.

We assume that all stocks have the same ratio of the traded volume to the market capitalization; we refer to this as "stock turnover" to distinguish it from the strategy turnover.[8] If we denote this constant stock turnover τ, we can write:

$$\mathrm{WAMC} = \frac{1}{\tau} V_P \sum_{S\in P} w_S v_S \quad (13)$$

A few insights from this model are apparent. First, turnover and WAMC have equal percentage contributions to the cost. Second, portfolio concentration affects the cost, even controlling for WAMC. A more concentrated portfolio is more expensive to trade. Finally, a strategy that tends to overweight low-volume stocks will be more expensive to trade.

Let us rewrite cost Equation (9) as follows:

$$C = k\frac{A}{\text{WAMC}}\left(T_a + T_d + \frac{1}{1-T_a}\psi_r T_r^2\right) \times \sum_{S\in P} w_S \frac{w_S}{v_S}\left(\frac{1}{\tau}\sum_{S\in P} w_S v_S\right) \quad (14)$$

Note that

$$\sum_{S\in P} w_S \frac{w_S}{v_S}\sum_{S\in P} w_S v_S = \sum_{S\in P} w_S^2 - cov^w\left(\frac{w_S}{v_S}, v_S\right) \quad (15)$$

and, therefore, we have the final expression for the cost:

$$C = \frac{k}{\tau}\frac{A\times ETO}{\text{WAMC}}(HI - LI) \quad (16)$$

Here, ETO is the effective turnover, $HI = \Sigma_{S\in P} w_S^2$ is the Herfindahl Index and $LI = cov^w(\frac{w_S}{v_S}, v_S)$ is a measure of the liquidity of the portfolio. The expression cov^w is the w_s-weighted covariance (equivalently, the covariance of two random variables where the probability of choosing a particular value equals w_s).

By assuming we have similar values of k, τ and A between any two strategies, proxied by k', and the effective turnover is proxied by replacement turnover (TO),[9] our simplified cost representation becomes

$$C = k'\frac{TO}{WAMC}(HI - LI) \quad (17)$$

A COMPARATIVE ANALYSIS

Using this framework, we compare the market impact cost of rebalancing nine portfolios: Cap 1000, Equal-Weight 1000 (with the same constituents as the Cap 1000) and Fundamental 1000 (an index whose constituent stocks are selected and weighted by an average of book, sales, dividends and income),[10] each in three regions: the United States, developed markets excluding the United States and emerging markets (EM). We initially assume identical strategy size and the same rebalance frequency. We do not include the market impact of net investment flows because they vary considerably depending on the nature and growth rate of the fund.

The index coverage is nearly identical because it is about 1.0 for a broad portfolio. Therefore, any difference in the market impact cost between the portfolios will be due to the base impact, effective turnover and tilt.

Given the same strategy size, the base impact is a function of the total trading volume of the universe. Exhibit 1 shows the snapshot at the end of June 2013. The effective turnover is, as noted earlier, composed of two components: a linear function for the additions and deletions, and a quadratic function for the reweighting of existing securities. Exhibit 2 displays the component values that emerged from our study.

Because fundamental size is more stable than market capitalization over time, the turnover from additions and deletions is smaller for the fundamental

EXHIBIT 1
Base Impact Snapshot as of June 30, 2013

Region	Volume USD Billions	Volume % of Global
United States	136	52%
Developed ex U.S.	96	36%
EM	31	12%
Global	264	100%

Source: Research Affiliates, LLC.

EXHIBIT 2
Components of Effective Turnover

Index	Component	U.S.	Dev. ex U.S.	EM
Cap 1000	$T_{adds\ \&\ deletes}$	3.9%	6.6%	11.7%
	$T_{reweightings}$	5.2%	5.7%	9.2%
	Effective turnover	4.2%	6.9%	12.4%
Fundamental 1000	$T_{adds\ \&\ deletes}$	3.1%	4.4%	8.4%
	$T_{reweightings}$	19.9%	22.3%	33.8%
	Effective turnover	9.2%	12.0%	25.2%
Equal-Weight 1000	$T_{adds\ \&\ deletes}$	17.4%	18.9%	27.2%
	$T_{reweightings}$	19.9%	19.6%	26.4%
	Effective turnover	22.9%	24.2%	36.2%

Source: Research Affiliates, LLC.

indexes than the cap-weighted indexes. Alternatively, the reweighting turnover is higher for the fundamental indexes because they require rebalancing against price movements. Equal-weighted indexes have the highest effective turnover due to the large number of trades for additions and deletions (those occur at larger weights in an equal-weighted index).

In virtually all cases, both turnover components are much higher in the emerging markets than in the developed regions; this is due in part to higher idiosyncratic volatility in the emerging markets.

Exhibit 3 summarizes the index tilt for the portfolios. Again, we note that the tilt increases dramatically as we move from the developed to the emerging markets region.

Finally, in Exhibit 4 we compare the aggregate measure of market impact (i.e., the product of the effective turnover, tilt and inverse universe volume). These numbers are scaled relative to the U.S. Cap 1000 portfolio. For example, the model predicts that, *at the same asset size*, the market impact cost of rebalancing a broad Fundamental U.S. index is almost three times greater than that of a broad U.S. cap-weighted index.

Of course, the assets tracking cap-weighted indexes (about $7 trillion by *Pensions & Investments* estimates)[11] are much greater than the fundamentals-weighted index assets (approximately $100 billion). Adjusted for that, it would seem that cap-weighted index investing has roughly 25 times as much market impact as fundamentals-weighted indexing. In reality, as the size of the cap-weighted strategies grew, new developments acted to mitigate the market impact cost. For example, index providers started to provide more lead time in pre-announcing index changes, thus encouraging liquidity providers to enter the market. Furthermore, as index portfolio managers became aware of the market impact, they presumably learned to trade less aggressively. These and similar factors are not accounted for in the market impact model presented here, but they are potential subjects for future research.

EXHIBIT 3
Index Tilt

Index	U.S.	Dev. ex U.S.	EM
Cap 1000	1.35	1.36	2.01
Fundamental 1000	1.66	1.39	1.71
Equal-Weight 1000	3.20	3.27	6.52

Source: Research Affiliates, LLC.

EXHIBIT 4
Market Impact Measures

Index	U.S.	Dev. ex U.S.	EM
Cap 1000	1.00	2.36	19.14
Fundamental 1000	2.68	4.19	33.01
Equal-Weight 1000	12.89	19.92	180.85

Source: Research Affiliates, LLC.

ENDNOTES

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[1]An index-based strategy trades at predefined reconstitution or rebalance dates, typically on an annual schedule but, in some cases, quarterly or monthly. The terms "reconstitution" and "rebalance" are often used interchangeably, and the distinction is unimportant for the purposes of this article. We shall henceforth use "rebalance." The target weights at each rebalance are determined at least a few days in advance.

[2]The variance attributable to explicit costs (i.e., observable price differences) is not always negative; a well-engineered implementation may do better than the underlying index.

[3]This is a reasonable assumption for an index trader who executes market-on-close, such as a swap dealer. A portfolio manager who spreads the trades over the course of the day will pay a weighted-average price somewhere between the original (pre-trade price) and the final (post-trade) price. If this weighted-average price is precisely halfway between the original and the final price, the cost impact would be reduced by half compared with our calculation.

[4]Turnover is admittedly a blunt measure. If the only trades at rebalance were divesting some stocks and replacing them with new ones (with the same total weight), the effective turnover would equal the actual turnover. In practice, however, much of the turnover comes from reweighting existing securities. Therefore, not all turnover is "equal." Suppose strategies A and B have the same turnover of 20%, but turnover due to additions and deletions represents the entire 20% for strategy A and only 5% for strategy B. The effective turnover for strategy A is 20%; for strategy B, it is

only 5% + 15%^{2} = 7.25%, or about one-third as much. This difference is not visible to portfolio analysts who rely exclusively on total turnover.

[5]Selecting the universe that corresponds to the cap-weighted benchmark would usually make the weight ratios more meaningful.

[6]This approximation assumes that prices follow a normal (not log-normal) distribution; clearly, it won't work well if the volatility is sufficiently high. In addition, the stocks that moved down in price by a large amount are more likely to be deleted, further lessening the precision of the estimate.

[7]Weighted-average market capitalization is defined $WAMC = \sum_{S \in P} w_S MC_S = MC_P \sum_{S \in P} w_S C_S$. Here, MC is the market capitalization (of either individual stocks s or the entire portfolio P), and c_s is the cap weight of stock s in portfolio P.

[8]In fact, we need the weaker but more complex assumption that the turnover is uncorrelated with certain weight-related measures; however, for our purposes the simpler one will do.

[9]Effective turnover will be bounded by turnover (TO) and the square of turnover (TO^2). In this case we are assuming that trades occur mainly due to replacement of securities rather than rebalancing across portfolios.

[10]See Arnott et al. [2005].

[11]See Zanona [2013].

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